Homework 6 – Answers to Questions

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**Question 1**:

Initial Matrix:

A =

Reflexive Closure (numbers changed are in red):

A’ =

Transitive closure pass 1:

A1 =

Transitive closure pass 2:

A2 =

Transitive closure pass 3:

A3 =

Transitive closure pass 4:

A4 =

Transitive closure pass 5:

A5 =

The final matrix represents the reflexive-transitive closure of the given relation. The matrix stops changing after the second pass but stops after the 5th pass because Warshall’s algorithm dictates that for each element in the set, (in this case there are 5 because it is a 5x5 matrix) there are that many steps in the transitive closure of the relation (in this case 5 steps because there are 5 elements.

**Question 2**:

Here is my final matrix after executing Floyd’s algorithm:

**Question 3**:

For the given matrix from problem 1, we get the unweighted directed graph:



There are 3 SCCs. The vertices a, b, and e form one, c forms the second, and d forms the third. Here is the condensation graph that fits that description:



One topological order is c, abe, d. Below is the adjacency matrix with the vertices in that order (top to bottom for the cols is c, abe, d and same for left to right for the rows):

Below is the matrix after performing the reflexive-transitive closure:

For a matrix to be a total order, it must show reflexivity, antisymmetry, and transitivity. The diagonal elements are all 1’s, so reflexivity satisfies. Total order wants antisymmetry where two distinct elements are mutually comparable. A total order is also known as a linear ordering. Based on where the 1s are placed, this condition, and therefore antisymmetry satisfies under total order. Lastly, for transitivity, whenever a b and b c, then a c should be true for all elements in the set. In the matrix whenever there is a 1 in positions (*i*, *j*) and (*j,* *k*), it also implies that *i* *k*. This holds for the above matrix. The matrix satisfies all three demanded properties, making it a total order.

**Question 4**:

Given Matrix:

The following matrices show the given matrix after each iteration of the outer loop. The values changed from the preceding matrix are in red:

D1 (intermediate vertex is 1) =

D2 (intermediate vertex is 2) =

D3 (intermediate vertex is 3) =

D4 (intermediate vertex is 4) =

**Resources:**

*Floyd-Warshall Algorithm*. (n.d.). Www.programiz.com. <https://www.programiz.com/dsa/floyd-warshall-algorithm>

Kwong, H. (n.d.). *7.4: Partial and Total Ordering* [Review of *7.4: Partial and Total Ordering*]. LibreTexts Mathematics. <https://math.libretexts.org/Bookshelves/Combinatorics_and_Discrete_Mathematics/A_Spiral_Workbook_for_Discrete_Mathematics_(Kwong)/07%3A_Relations/7.04%3A_Partial_and_Total_Ordering>

Singh, J. (2021). Reflexive Closure [YouTube *Closure of Relations – Part 1*]. In *Neso Academy*. <https://www.youtube.com/watch?v=SgQJlKLWJmY>

Singh, J. (2021). Warshall’s Algorithm (Finding the Transitive Closure) [YouTube *Warshall’s Algorithm (Finding the Transitive Closure)*]. In *Neso Academy*. <https://www.youtube.com/watch?v=_-p8zhizock&t=3s>

*what is total order - explanation please*. (n.d.). Mathematics Stack Exchange. Retrieved December 6, 2023, from <https://math.stackexchange.com/questions/239118/what-is-total-order-explanation-please>